Key

Dimensional analysis practice problems
An important skill to learn for science/engineering courses.

INSTRUCTIONS: Use dimensional analysis for all of the following problems. Show your work (show which units cancel by crossing them out). Use **significant figures** for your final answer. <u>*Remember, you are always responsible for knowing metric to metric conversions, but you do not need to memorize metric to English conversions.*</u>

REMEMBER: THE PROBLEMS ARE EASY ONCE YOU FIGURE OUT WHAT INFORMATION TO START WITH.

1. When one gram of gasoline burns in a car's engine, the amount of energy given off is approximately 1.03×10^4 cal. Express this quantity in joules (J). (Use 1 cal = 4.184 J)

$$1.03 \times 10^4 \cancel{cal} \times \left(\frac{4.184\,J}{1 \cancel{cal}}\right) = 43095 \rightarrow \boxed{4.31 \times 10^4\,J}$$

2. The pressure reading from a barometer is 742 mm Hg. Express this reading in kilopascals, kPa. (Use 760 mm Hg = 1.013×10^5 Pa)

$$742 \cancel{mm\,Hg} \times \left(\frac{1.013 \times 10^5 \cancel{Pa}}{760 \cancel{mm\,Hg}}\right) \times \left(\frac{1\,kPa}{1000 \cancel{Pa}}\right) = \boxed{98.9\,kPa}$$

3. How many megayears is equivalent to 6.02×10^{23} nanoseconds (ns)?

$$6.02 \times 10^{23} \cancel{ns} \times \left(\frac{1 \cancel{s}}{10^9 \cancel{ns}}\right)\left(\frac{1 \cancel{min}}{60 \cancel{s}}\right)\left(\frac{1 \cancel{hr}}{60 \cancel{min}}\right)\left(\frac{\cancel{day}}{24 \cancel{hr}}\right)\left(\frac{1 \cancel{yr}}{365 \cancel{days}}\right)\left(\frac{1\,megayr}{10^6 \cancel{yr}}\right) = \boxed{19.1\,My}$$

not accounting for leap yr

4. The average student is in class 330 min/day.

a. How many hours/day is the average student in class? *You should know what conversion factors you will need!*

$$\frac{330 \cancel{min}}{day} \cdot \left(\frac{hr}{60 \cancel{min}}\right) = \boxed{5.5\,hr/day}$$

b. How many seconds is the average student in class per week?

$$1 \cancel{week} \times \left(\frac{5 \cancel{days\,of\,school}}{1 \cancel{week}}\right) \times \left(\frac{5.5 \cancel{hr}}{1 \cancel{day}}\right) \times \left(\frac{60 \cancel{min}}{1 \cancel{hr}}\right) \times \left(\frac{60\,s}{1 \cancel{min}}\right) = \boxed{9.9 \times 10^4\,s}$$

c. Calculate how many seconds you are in class a day (pick any weekday you have class). Show any calculations you have to do as dimensional analysis below. *Answers will vary between you and your classmates.*

<u>answers will vary.</u>

5. Approximately how many pounds do you weigh? 150 lbs.

The human body is *approximately* 60% water by mass. You can write this as a conversion factor: 60g water / 100g body mass. (Does this make sense?)

you can use lb. instead

Using that conversion factor, <u>how many pounds of water are there in your body</u>? (This is a rough estimate, since it also depends on body fat).

$$150 \cancel{\text{lb body}} \times \left(\frac{60 \text{ lb water}}{100 \cancel{\text{lb body}}}\right) = \boxed{90 \text{ lb water}}$$

full label, so you don't confuse lb water vs lb body

6. French cooks usually weigh ingredients. A French recipe uses 225 grams of granulated sugar. How many cups are needed if there are 2 cups of sugar per pound of sugar? Round to the nearest cup.

(Note that you are changing from units of weight, grams, to units of volume, cups. The conversion factor 2 cups per pound is true for sugar but may not be the same for all other ingredients.)

(1 lb = 453.6g)

$$225 \cancel{g} \times \left(\frac{1 \cancel{lb}}{453.6 \cancel{g}}\right) \times \left(\frac{2 \text{ cups}}{\cancel{lb}}\right) = 0.99 \text{ cups} \left(\text{or } 1 \text{ cup}\right)$$

(How many sig figs would you use? In real life, would you use 0.99 cups or 1 cup?)

7. Convert 10.3 g · cm^3/second^2 to kg · m^3 /hour^2

(· means multiplication, so both g and cm^3 are in the numerator; seconds is abbreviated "s" and hour is "h") Use conversion factors that you already know (or *should* know!)

$$\frac{10.3 \cancel{g} \cdot \cancel{cm^3}}{\cancel{s^2}} \times \left(\frac{kg}{1000 \cancel{g}}\right) \times \left(\frac{m^3}{100^3 \cancel{cm^3}}\right) \times \left(\frac{60^2 \, min^2}{1 \, hr^2}\right)\left(\frac{60^2 \, s^2}{1 \, min^2}\right) = \boxed{0.134 \frac{kgm^2}{hr^2}}$$

8. A car accelerates at 12 mi/hr·s (miles per hour-seconds). Write that acceleration in m/s^2.

(1 mile = 1.609 km)

$$\frac{12 \cancel{mi}}{\cancel{hr} \cdot s} \times \left(\frac{1.609 \cancel{km}}{1 \cancel{mi}}\right) \times \left(\frac{1000 \, m}{1 \cancel{km}}\right) \times \left(\frac{1 \, hr}{60 \, min}\right) \times \left(\frac{1 \, min}{60 \, s}\right) = \boxed{5.4 \, m/s^2}$$

9. Because your 18 year-old friend never learned dimensional analysis, he started working at a fast food restaurant wrapping hamburgers. Every 3 hours he wraps 350 hamburgers. He works 8 hours per day. He works 5 days a week. He gets paid every 2 weeks with a salary of $440.34 (after taxes!).

a. Approximately how many hamburgers will he have to wrap to make his first one million dollars? (If you can solve the problem, you know dimensional analysis and maybe you can teach your friend so that he can get a better job!) This will be an estimate, so round to 1 significant figure. Explicitly show all conversions in your setup.

$$\$1\times10^6 \times \left(\frac{2\text{ wks}}{\$440.34}\right)\left(\frac{5\text{ days}}{1\text{ wk}}\right)\left(\frac{8\text{ hr}}{1\text{ day}}\right)\left(\frac{350\text{ hamburgers}}{3\text{ hr}}\right) = 21195743 \rightarrow \boxed{2\times10^7\text{ burgers}}$$

a million dollars

b. How much time will it take to wrap all those hamburgers (given as your answer to part a)? **Express the time in years**. Assume that he will work 40 weeks out of the year and that he works with the same efficiency everyday. By the way, be careful on this one—no one works 24 hours a day! Round your answer to the nearest year.

$$21195743\text{ burgers} \times \left(\frac{3\text{ hr}}{350\text{ burgers}}\right)\left(\frac{\text{day}}{8\text{ hr}}\right)\left(\frac{1\text{ wk}}{5\text{ days}}\right)\left(\frac{1\text{ yr}}{40\text{ wk}}\right) = \boxed{114\text{ yrs!}}$$

c. How old will he be when he makes his first million? (Let's hope he makes it to this ripe old age!)

$$114\text{ yrs} + 18\text{ yrs} = \boxed{132\text{ yrs old}}$$

10. A child is entered into the hospital after ingesting 12 aspirin tablets. The Merck Index indicates that renal (kidney) failure can occur if as little as 3 grams is ingested, and may be fatal if as much as 10 grams is eaten. If each aspirin tablet contains 300 mg of aspirin, how much aspirin (in grams) has the child ingested?

$$12\text{ tablets} \times \left(\frac{300\text{ mg}}{1\text{ tablet}}\right)\left(\frac{\text{g}}{1000\text{ mg}}\right) = 4\text{ g}$$

exact # do not count as 2 SFs

renal failure will probably occur. !!

11. A patient in the hospital is given an intravenous fluid that must deliver 1000 cc of a dextrose (sugar) solution over 8 hours. The intravenous fluid tubing delivers 15 drops/cc. What is the drop rate (in units of drops/min) that must be administered to the patient? Round the answer to 2 significant figures.
(Memorize and use this: 1 cc = 1 cm^3 = 1 mL).

$$\frac{15\text{ drops}}{\cancel{\text{cc}}} \times \left(\frac{1000\ \cancel{\text{cc}}}{8\ \cancel{\text{hrs}}}\right) \times \left(\frac{1\ \cancel{\text{hr}}}{60\text{ min}}\right) = \boxed{31\text{ drops/min}}$$

12. Different situation. A medical doctor gives the order to administer dopamine at a rate of 3.0 mcg / kg·min (mcg is the abbreviation for microgram in a medical context). The dopamine is supplied as a mixture of 400. mg dopamine in 250. mL of a dopamine solution. The patient weighs 73 kg. What is the infusion rate of the dopamine into her body (in units of mL/hour)?

$$73\ \cancel{\text{kg}} \cdot \left(\frac{3.0\ \cancel{\text{mcg}}}{\cancel{\text{kg}}\ \cancel{\text{min}}}\right)\left(\frac{1\ \cancel{\text{mg}}}{1000\ \cancel{\text{mcg}}}\right)\left(\frac{250.\text{ mL}}{400\ \cancel{\text{mg}}}\right)\left(\frac{60\ \cancel{\text{min}}}{1\text{ hr}}\right) = \boxed{\frac{8.2\text{ ml}}{\text{hr}}}$$

13. Analysis of an air sample reveals that it contains 3.5 x 10^{-6} g/L of carbon monoxide. Express the concentration of carbon monoxide in lb/ft^3. (Use 1.00 lb = 454 g; 1 in = 2.54 cm)

$$\frac{3.5\times10^{-6}\ \cancel{\text{g}}}{\cancel{\text{L}}} \cdot \left(\frac{\text{lb}}{454\ \cancel{\text{g}}}\right) \cdot \left(\frac{1\ \cancel{\text{L}}}{1000\ \cancel{\text{cm}^3}}\right)\left(\frac{2.54^3\ \cancel{\text{cm}^3}}{1\ \cancel{\text{in}^3}}\right) \cdot \left(\frac{12^3\ \cancel{\text{in}^3}}{1\text{ ft}^3}\right) = \boxed{\frac{2.2\times10^{-7}\text{ lb}}{\text{ft}^3}}$$

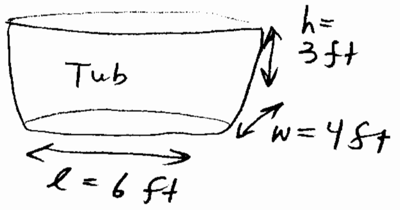
14. A website stated that the average body density is 0.001 kg/cm^3. Is this the same as 62 lbs/ft^3? *(1 inch = 2.540 cm)*

$$\frac{62\text{ lbs}}{\text{ft}^3} \cdot \left(\frac{454\text{ g}}{1\text{ lb}}\right) \cdot \left(\frac{1\text{ kg}}{1000\text{ g}}\right) \cdot \left(\frac{1\text{ ft}^3}{12^3\text{ in}^3}\right) \cdot \left(\frac{1\text{ in}^3}{2.54^3\text{ cm}^3}\right) = \boxed{9.9\times10^{-4}\text{ is very close to } 0.001\text{ kg/cm}^3}$$

b. By the way, if the average body density is 62 lbs/ft^3, what does it mean (in terms of fat vs muscle) if your body density is less than 62 lbs/ft^3? What if it is more than 62 lbs/ft^3?

Fat is less dense than muscle,
so less than 62 lb/ft^3 means more fat
more than 62 lb/ft^3 means more muscle
than the average body.

15. The bathtub in the residential suite of the White House had to be enlarged for Warren G. Harding (WGH). (Dr. Atkins, where were you when he needed you!?!?!) WGH weighed 370. pounds. The bathtub is four feet wide, six feet long, and three feet high. How many gallons of water are needed to fill the WGH Memorial Bathtub up to the rim? (1 foot = 12 inches; 1 gal = 231 in^3). [We won't even talk about President Taft's tub...]



$$72\,\cancel{ft^3} \times \left(\frac{12^3\,\cancel{in^3}}{1\,\cancel{ft^3}}\right)\left(\frac{1\text{ gal}}{231\,\cancel{in^3}}\right) = 538\ldots \rightarrow \boxed{540\text{ gal.}}$$

$$V = l \times w \times h = 72\text{ ft}^3$$

b. Well, that's sort of a wacky question...why would WGH get in a tub completely filled with water? (I guess HE wouldn't have to clean it up...) The average body density is 62 lbs/ft^3. How many gallons of water are needed to fill the tub so that when WGH gets in the tub, the water reaches the rim but does not flow over? *(0.4536 kg = 1 lb)*

$$V_{tub} - V_{WGH} = V_{water} \longrightarrow 72\text{ ft}^3 - 5.97\text{ ft}^3 = 66\text{ ft}^3$$

$$V_{WGH} = 370.\text{ lb} \times \left(\frac{ft^3}{62\text{ lb}}\right) = 5.97\text{ ft}^3$$

$$66\text{ ft}^3 \times \left(\frac{12^3\text{ in}^3}{1\text{ ft}^3}\right) \times \left(\frac{1\text{ gal}}{231\text{ in}^3}\right) = \boxed{490\text{ gal}}$$

16. Albumin is a protein found in blood. If the concentration of this protein is 600. micromole per liter, and its molecular mass is 68,500. g/mol, what is the concentration of this protein in units of milligrams per cubic centimeter?

$$\frac{600.\,\cancel{\mu mol}}{\cancel{L}}\left(\frac{\cancel{mol}}{10^6\,\cancel{\mu mol}}\right)\left(\frac{68500\,\cancel{g}}{\cancel{mol}}\right)\left(\frac{1000\text{ mg}}{1\,\cancel{g}}\right)\left(\frac{1\,\cancel{L}}{1000\text{ cm}^3}\right) =$$

$$\boxed{41.1\text{ mg/cm}^3}$$

17. So you escaped the fate of wrapping hamburgers. The next step is to continue proving that you know dimensional analysis (it's never too late for the hamburger people to come get you!) The great thing about dimensional analysis is that sometimes you can solve problems without equations. Here is such a case:

a. All matter has a property called a specific heat capacity. For silver, this specific heat capacity is 0.24 J/°C · g. How much energy (in Joules) would be required to heat 120.0 g of silver (Ag) so that its temperature changes by 32°C? Use dimensional analysis, not an equation!

$$\frac{0.24\text{ J}}{\cancel{°C}\cdot\cancel{g}} \times \left(\frac{120.0\,\cancel{g}}{1}\right) \times \left(\frac{32\,\cancel{°C}}{1}\right) = 921.6 \rightarrow \boxed{920\text{ J}}$$

b. Based on how you set up the problem above, what would be the **equation**? Fill in the rest of this expression to form your own equation (that you figured out by using dimensional analysis above.) You will use the terms "**mass**" "**specific heat**" and "**temperature**" and some mathematical operation signs). This answer is an equation, not a dimensional analysis setup!

Energy (J) = mass * specific heat * temperature

Proof: $J = \cancel{g} \cdot \left(\frac{J}{\cancel{g} \cdot \cancel{°C}} \right) \cdot \cancel{°C}$

18. The balanced reaction of sulfuric acid with sodium hydroxide is shown below:

H_2SO_4 + 2 NaOH ⟶ Na_2SO_4 + 2 H_2O

For 146 grams of H_2SO_4, how many grams of H_2O can be made (assume you have all the NaOH you need for a complete reaction)?

Use dimensional analysis to solve this problem. **Circle the conversion factors you need from the list, then show the dimensional analysis to solve the problem.**

List of possible conversion factors (you only need some of these to solve the problem):

1 mole H_2SO_4 = 1 mole Na_2SO_4

1 mole H_2SO_4 = 2 mole NaOH

1 mole H_2SO_4 = 2 mole H_2O

1 mole Na_2SO_4 = 2 mole H_2O

1 mole H_2SO_4 = 98.07 g H_2SO_4

1 mole NaOH= 40.01 g NaOH

1 mole Na_2SO_4= 142.04 g Na_2SO_4

1 mole H_2O= 18.02 g H_2O

$$146\text{g } H_2SO_4 \left(\frac{1 \text{ mol } H_2SO_4}{98.07\text{g } H_2SO_4} \right) \cdot \left(\frac{2 \text{ mol } H_2O}{1 \text{ mol } H_2SO_4} \right) \cdot \left(\frac{18.02\text{ g } H_2O}{1 \text{ mol } H_2O} \right) = \boxed{53.7\text{g } H_2O}$$